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Unstable particles and the Poincaré semigroup in quantum field theory

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Abstract. We propose to describe the dynamics of unstable particles in relativistic quantum field theory in terms of semigroups of transformations of the observables. This leads, in contrast to the usual Hilbert space level treatment, to a complete and consistent description of the irreversible dynamics of decay processes. The scheme is explicitly worked out for the massive scalar quantum field and the evolution of the particle density and its higher moments is computed.

1. Introduction

Theories of unstable particles still face many conceptual difficulties. First of all, unstable particles represent an irreversible process (decay process) which as with each irreversible phenomenon asks for a microscopic explanation. In this paper we are not focusing on the derivation of the phenomenon of irreversibility hidden in the theory of unstable particles, but we accept the irreversibility as such and try to describe it in a way as has been suggested by many authors over the last fifteen years [1-4]. The essential idea is that the state of an unstable particle cannot move backwards in time. The transformations which can be applied can at best be the representatives of a semigroup constituted by the future cone of Minkowski space.

Although this idea is widely known, the representations of the future cone which one finds in the literature are only realised on the one-particle space and are not extended to the many-particle states, hence excluding essentially the field theoretic character.

Our contribution now precisely consists of the construction of representations of the future cone by semigroups taking into account the many-particle structure of quantum field theory. In particular we use the theory of completely positive unity preserving maps recently introduced in statistical mechanics of irreversible systems (for reviews of the theory see [5-7]). Such mappings induce a transformation of the states of the system into themselves and therefore allow a probabilistic interpretation of the theory. In general pure states are mapped into mixed states; this is a characteristic property for irreversibility, which therefore cannot be realised on the level of Hilbert space transformations. We work out the details for the massive scalar field. On the one-particle level one recovers the known results, but our theory enables us, for example, to calculate also the moments of the particle number operator in an n-particle state.

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Hence having done this we obtain a genuine quantum field theory of unstable particles which contains the essential ingredients of irreversibility, Poincaré covariance and statistical independence of decay processes and which is compatible with the probabilistic interpretation of quantum theories.

2. Quantum irreversible dynamics and relativistic unstable particles

First we describe non-relativistic quantum irreversible dynamics in a theory of open systems [5-7].

Consider a system with algebra of observables $\mathscr{B}(\mathscr{H})$, all bounded operators on a Hilbert space $\mathscr{H}$. As states of the system we consider the expectation functionals defined by means of density matrices ρ on $\mathscr{H}$:

$$\rho(A)=\operatorname{Tr}\rho A, \qquad A\in\mathscr{B}(\mathscr{H}).$$

Here we use the same notation for the state and for the corresponding density matrix.

In the Heisenberg picture the irreversible evolution from time $t=0$ up to time t is given by the dynamical map Γ_t:

$$\Gamma_t: A\in\mathscr{B}(\mathscr{H})\rightarrow\Gamma_t(A)\in\mathscr{B}(\mathscr{H}) \tag{1}$$

satisfying

(1) Γ_t is a linear map of $\mathscr{B}(\mathscr{H})$

(2) $\Gamma_t(1)=1$ where 1 stands for the unit operator in $\mathscr{B}(\mathscr{H})$

(3) Γ_t is a completely positive map, i.e. for all $n\in\mathbb{N}$, the map $\Gamma_t\otimes 1$ of $\mathscr{B}(\mathscr{H})\otimes M_n$ (M_n the set of $n\times n$ matrices) is a positive map.

Such a dynamical map can be described in the Schrödinger picture (mapping states into states) by the transposed map $\hat{\Gamma}_t$ of Γ_t defined as follows:

$$\hat{\Gamma}_t(\rho)=\rho\cdot\Gamma_t.$$

In general such dynamical maps $\hat{\Gamma}_t$ map pure states into mixed states and hence describe an irreversible evolution containing some stochasticity, due to the environment.

In general the time dependence may be very complicated. However here we restrict ourselves to the Markovian case:

$$\Gamma_0=1, \qquad \Gamma_{t_1}\Gamma_{t_2}=\Gamma_{t_1+t_2}, \qquad t_1, t_2\in\mathbb{R}^+ \tag{2}$$

i.e. $\{\Gamma_t|t\in\mathbb{R}^+\}$ is a representation of the semigroup $\mathbb{R}^+$ into a one-parameter semigroup of unity preserving completely positive maps of $\mathscr{B}(\mathscr{H})$.

Under the uniform continuity condition of the map $\Gamma: t\in\mathbb{R}^+\rightarrow\Gamma_t$ one has that there exists a generator L such that

$$\Gamma_t=\exp tL$$

where L is of the Lindblad type [8], i.e. for all $A\in\mathscr{B}(\mathscr{H})$:

$$L(A)=\mathrm{i}[H, A]+\tfrac{1}{2}\sum_k\{V_k^*[A, V_k]+[V_k^*, A]V_k\} \tag{3}$$

where $H, V_k\in\mathscr{B}(\mathscr{H})$ and $H^*=H$.

Here we formulated the notion of a dynamical semigroup acting on $\mathscr{B}(\mathscr{H})$. It is clear that this can be generalised to the more general scheme of a physical system given by a C^*-algebra of observables $\mathscr{A}$ and its corresponding set of states (the positive linear functionals of $\mathscr{A}$ which are normalised up to one). Moreover one may assume weaker continuity conditions.

In many cases one is also able to construct a generator for these semigroups. They are formally still of the type (3), but the operators H and V_k are generally unbounded and instead of a discrete sum one might have integrals.

Physically, irreversibility should be due to an interaction of the system with its environment. Indeed for a large class of dynamical semigroups one can construct a reversible (unitary) evolution for a larger system. The semigroup can then be recovered as a reduced dynamics (dilation construction) or as a Markovian limit (weak coupling or singular coupling limits).

Our aim is now to generalise the above scheme to relativistic theories. These are theories which are covariant for the Poincaré group $\mathscr{P}$ consisting of all pairs (a, L) with $a \in \mathbb{R}^4$ and $L \in \mathscr{L}$, the proper orthochronous Lorentz group. The Poincaré group acts on the Minkowski space $\mathbb{R}^4$ equipped with the metric

$$(x, y) = x_0 y_0 - \bar{x} \cdot \bar{y}$$

as a transformation group in the following way:

$$(a, L)x = Lx + a.$$

The product rule in $\mathscr{P}$ is then

$$(a, L)(b, M) = (a + Lb, LM)$$

for (a, L), $(b, M) \in \mathscr{P}$.

In order to describe a non-relativistic irreversible evolution one needs a representation of the time-parameter semigroup $\mathbb{R}^+$ into the dynamical semigroups of the system; $\mathbb{R}^+$ represents the future part of the absolute time variable.

In relativistic theories this is replaced by the future cone

$$\mathscr{F} = \{a \in \mathbb{R}^4 | a^2 = (a, a) \geq 0, a_0 \geq 0\}. \tag{4}$$

We remark that $\mathscr{F}$ is an additive semigroup, a sub-semigroup of $\mathbb{R}^4$.

Here also one describes relativistic systems by means of the so-called algebraic approach [9]. A dynamical system has as fundamental ingredients the algebra of observables given by a C^*-algebra $\mathscr{A}$ with unit, the physical states of the system given by the positive linear normalised functionals ω of $\mathscr{A}$ and the kinematics given by a representation π of the Poincaré group into the *-automorphisms of $\mathscr{A}$.

Now we propose to describe the dynamics of unstable particles by a Poincaré covariant representation Γ of the future cone $\mathscr{F}$ into a semigroup of linear completely positive unity preserving maps of $\mathscr{A}$, i.e. for all $b \in \mathscr{F}$, Γ_b is a linear completely positive unity preserving map of $\mathscr{A}$, satisfying the semigroup property

$$\Gamma_a \Gamma_b = \Gamma_{a+b}, \qquad a, b \in \mathscr{F} \tag{5}$$

and the covariance

$$\pi(a, L)\Gamma_b \pi(a, L)^{-1} = \Gamma_{Lb} \tag{6}$$

for all $(a, L) \in \mathscr{P}$ and $b \in \mathscr{F}$.

In the next section we implement this scheme for a scalar field of particles with a non-vanishing mass.

Before closing this section we want to comment on the microscopic origin of these relativistic semigroups.

One may start by choosing a particular spacetime direction $(t, 0)$, $t > 0$ in $\mathscr{F}$. Afterwards one recovers any direction by applying the Poincaré group.

Then one should obtain a dynamical semigroup $\Gamma_t = \Gamma_{(t,0)}$ as a Markovian approximation for the reduced dynamics of an open system (the unstable particles) interacting with a reservoir (decay products). Technically this is not an easy feat. Because of the non-existence of relativistic dynamics, one introduces cutoffs and smooth form factors. Then one proceeds as in the non-relativistic case with a weak coupling limit [10]. Afterwards one may formally remove the cutoffs to obtain a covariant dynamical semigroup.

3. Scalar boson field

Consider the Hilbert space $H = L^2(\mathbb{R}^4, d\mu_m)$ of square integrable complex functions on $\mathbb{R}^4$ with the usual Lorentz invariant measure $d\mu_m = \delta(p^2 - m^2)\theta(p_0)\,dp$ on the hyperboloid S_m of positive energy with mass m. This is the relativistic test-function space of the boson field described by the CCR-algebra $\mathscr{A}(\sigma)$. The latter is generated by the Weyl operators $W(\phi)$, $\phi \in \mathrm{H}$, satisfying the product rule:

$$W(\phi)\,W(\psi) = \exp[-\tfrac{1}{2}\mathrm{i}\sigma(\phi, \psi)]\,W(\phi + \psi)$$

where $\sigma(\phi, \psi) = \mathrm{Im}(\phi, \psi)$ and $W(\phi)^* = W(-\phi)$. One might think of the Weyl operators as explicitly given by

$$W(\phi) = \exp \mathrm{i}\left(\frac{(a(\phi) + a^*(\phi))}{\sqrt{2}}\right)$$

where $a^*(\phi)$, $a(\phi)$ are the usual creation and annihilation operators on Fock space.

By the construction of H, as the measure $d\mu_m$ is Lorentz invariant, we have a natural unitary representation U of the Poincaré group $\mathscr{P}$ on the test-function space H

$$(U(a, L)\phi)(p) = \exp(\mathrm{i}ap)\phi(L^{-1}p). \tag{7}$$

It is easily checked that this representation U induces a representation π of $\mathscr{P}$ into the *-automorphisms of $\mathscr{A}(\sigma)$:

$$\pi(a, L)\,W(\phi) = W(U(a, L)\phi), \qquad (a, L) \in \mathscr{P}. \tag{8}$$

This is the representation π of the kinematics mentioned above. We remark that the maps $\pi(a, L)$ map Weyl operators into Weyl operators. Such automorphisms are called quasi-free.

Now we are looking for a representation Γ of the semigroup $\mathscr{F}$ also into quasi-free maps. Quasi-free completely positive unity preserving maps of the CCR-algebra $\mathscr{A}(\sigma)$ into itself have been extensively studied before [11] and shown to be of the form

$$W(\phi) \to W(A\phi)f(\phi) \tag{9}$$

where A is a linear operator on H and where f is any complex function such that

$$W(\phi) \to f(\phi)$$

is a quasi-free state [12] on the modified CCR-algebra $\mathscr{A}(\sigma_A)$ where $\sigma_A(\phi, \psi) = \sigma(\phi, \psi) - \sigma(A\phi, A\psi)$, $(\phi, \psi \in H)$.

Therefore we consider the following representation of the semigroup

$$b \to \Gamma_b \qquad b \in \mathscr{F}$$

where

$$\Gamma_b W(\phi) = W(T_b\phi)\exp(\mathrm{i}\sigma(\chi_b, \phi))\exp(-\tfrac{1}{4}\langle\phi|X_b\phi\rangle)$$

where T_b, X_b are bounded linear operators on H and χ_b is a vector in H such that

(a) $\langle\phi|X_b\phi\rangle \geq 0\ \forall\phi\in H$

(b) $|\sigma_{T_b}(\phi, \psi)|^2 \leq \langle\phi|X_b\phi\rangle\langle\psi|X_b\psi\rangle$; see [12]

(c) the maps $b\in\mathscr{F}\to T_b$, χ_b and X_b are weakly continuous.

Our aim is now to obtain the explicit forms of χ_b, T_b and X_b using the condition that Γ_b is a Poincaré covariant semigroup in the sense of [5, 6].

Theorem. With the above notation $\{\Gamma_b|b\in\mathscr{F}\}$ is a Poincaré covariant semigroup of unity preserving completely positive mappings iff

(i) $\exists z\in\mathbb{C}$, $\operatorname{Im} z\geq 0$, such that

$$(T_b\phi)(p) = \exp(\mathrm{i}zbp)\phi(p), \qquad \phi\in H$$

(ii) $\chi_b = 0$

(iii) $\exists\kappa\geq 1$ such that

$$(X_b\phi)(p) = \kappa[1-\exp(-2\operatorname{Im} zbp)]\phi(p), \qquad \phi\in H.$$

Proof. By inspection the conditions (i), (ii) and (iii) imply that $\{\Gamma_b|b\in\mathscr{F}\}$ is a Poincaré covariant semigroup. We now prove the converse statement. From the Poincaré covariance it follows immediately that

$$U(a, L)T_bU(a, L)^{-1} = T_{Lb}$$

$$U(a, L)\chi_b = \chi_{Lb}$$

$$U(a, L)X_bU(a, L)^{-1} = X_{Lb}.$$

Putting $L=1$ we find that T_b and X_b are invariant with respect to the translations and therefore there exist bounded functions $T_b(p)$ and $X_b(p)$ on the hyperboloid S_m such that

$$\left.\begin{aligned}(T_b\phi)(p) &= T_b(p)\phi(p)\\ (X_b\phi)(p) &= X_b(p)\phi(p)\end{aligned}\right\}\qquad \phi\in H.$$

The vector $\chi_b\in H$ is translation invariant:

$$\chi_b = 0.$$

Now putting $a=0$ one concludes the Lorentz covariance

$$T_b(L^{-1}p) = T_{Lb}(p) \qquad X_b(L^{-1}p) = X_{Lb}(p).$$

On the other hand as Γ_b is a semigroup we have

$$T_b(p)T_{b'}(p) = T_{b+b'}(p), \qquad T_0(p) = 1$$

and

$$X_{b+b'}(p) = |T_{b'}(p)|^2X_b(p) + X_{b'}(p), \qquad X_0(p) = 0.$$

From this and the continuity $b\to T_b$ it follows that there exists a vector field $\gamma(p)$ such that

$$T_b(p) = \exp b\gamma(p).$$

From the Lorentz covariance and the fact that T_b is a bounded linear operator, there exists $z \in \mathbb{C}$, $\operatorname{Im} z \geqslant 0$, such that

$$T_b(p) = \exp \mathrm{i}zbp.$$

For the operator X_b the semigroup condition allows us to write

$$X_{b+b'}(p) = |T_{b'}(p)|^2 X_b(p) + X_{b'}(p) = |T_b(p)|^2 X_{b'}(p) + X_b(p).$$

Therefore there exists a function $\kappa(p)$ independent of b such that

$$\frac{X_b(p)}{1 - |T_b(p)|^2} = \kappa(p)$$

and using the Lorentz covariance one concludes that κ has to be a constant. Finally we should satisfy the positivity conditions (a) and (b) in the definition of the quasi-free completely positive map Γ_b which imply $\kappa \geqslant 1$.

Next we give a few comments on the theorem and its possible generalisations. The contraction semigroup T_b acting on H describes the unstable particle on the level of a one-particle space. Such a description on the Hilbert space level has frequently been used in the literature [1-4]. The main point of our theorem is, however, that the description of unstable particles should be done on the quantum field level rather than on the level of a single-particle Hilbert space. Clearly $\gamma \equiv 2 \operatorname{Im} z$ determines the decay rate of the unstable particles and in the following we choose $\operatorname{Re} z = 1$ in order that Γ_b coincides with $\pi(b, 1)$ in the limiting case $\gamma = 0$.

By inspection $\omega_0(W(\phi)) = \exp(-\frac{1}{4}\kappa \|\phi\|^2)$ is the only invariant regular state under $\{\Gamma_b | b \in \mathscr{F}\}$ and all regular states evolve, when $b_0 \to \infty$, under Γ_b to ω_0. The state ω_0 is pure iff $\kappa = 1$ [12] and in this case it is the usual Fock vacuum ω_{F}. If $\kappa > 1$ we have a mixed state which describes a background density of particles. From now on we will restrict ourselves to the Fock case.

Let $a(\phi)$ and $a^*(\phi)$ denote the usual boson field operators on Fock space satisfying

$$[a(\phi), a(\psi)] = 0 \qquad \text{and} \qquad [a(\phi), a^*(\psi)] = \langle \phi | \psi \rangle.$$

The action of Γ_b on a Fock space operator can then formally be written as

$$\Gamma_b = \exp bL$$

where

$$L(x) = \int \mathrm{d}\mu_m(p) p \{\mathrm{i}[a^*(p)a(p), x] + \tfrac{1}{2}\gamma(a^*(p)[x, a(p)] + [a^*(p), x]a(p))\}.$$

So L has the formal structure of a Lindblad generator (3). One can also easily compute the action of Γ_b on any monomial in the field operators and obtain

$$\begin{aligned}\Gamma_b(a^{\#}(\phi_1) \ldots a^{\#}(\phi_n)) &= \sum_{\pi} a^{\#}(T_b\phi_{i_1}) \ldots a^{\#}(T_b\phi_{i_k}) \\ &\quad \times \omega_{\mathrm{F}}(a^{\#}(\{1 - |T_b|^2\}^{1/2}\phi_{i_{k+1}}) \ldots a^{\#}(\{1 - |T_b|^2\}^{1/2}\phi_{i_n}))\end{aligned} \qquad (10)$$

where the summation is on all partitions of $\{1, \ldots, n\}$ into subsets $\{i_1, \ldots, i_k\}$ $\{i_{k+1}, \ldots, i_n\}$ such that $i_1 \leqslant \ldots \leqslant i_k$ and $i_{k+1} \leqslant \ldots \leqslant i_n$ and where $a^{\#}$ means either a or a^*.

The construction of the above equation can be generalised to the case where the CCR-algebra is built on a dense subspace of H such as the Schwartz space. In principle this allows generalisation of the function $\phi \to \sigma(\psi_b|\phi)$ to a distribution and of X_b to an unbounded operator. However due to the Poincaré covariance and the non-vanishing mass of the particles the Poincaré covariant semigroups still coincide with the ones obtained in the theorem.

For the mass m scalar field only the one-dimensional representations of the Lorentz group enter in the construction. For spin non-zero fields with possible internal degrees of freedom higher-dimensional representations of the restricted Lorentz group should be taken into account, e.g. the semigroup Γ_b should be suitably generalised in the sense that the semigroup $\{T_b|b \in \mathscr{F}\}$ is a non-trivial matrix representation of the semigroup $\mathscr{F}$.

In this work we have focused on Bose fields but the same programme can be carried out for Fermi fields. In this case one has to take into account the above remark and use the following form of the semigroup [13, 14]

$$\begin{aligned}\Gamma_b(a^{\#}(\phi_1)\ldots a^{\#}(\phi_n)) &\\ &= \sum_{\pi} \varepsilon_\pi a^{\#}(T_b\phi_{i_1})\ldots a^{\#}(T_b\phi_{i_k})\\ &\quad \times \omega_{\mathrm{F}}((a^{\#}(1-T_b^*T_b)^{1/2}\phi_{i_{k+1}})\ldots a^{\#}((1-T_b^*T_b)^{1/2}\phi_{i_n}))\end{aligned}$$

where ε_π is the index of the permutation $\binom{1\ldots n}{i_1\ldots i_n}$.

4. Applications

4.1. Decay of moving particles

In the problem of the decay of moving particles one is measuring the density of a beam of particles which move with mean velocity $\bar{v}$ with respect to the laboratory frame, i.e. we compare the density of particles $n(p;0)$ with the energy momentum $p=(p_0,\bar{p})$ at the origin $(0,0)$ with the density $n(p,b)$ at $b=(t,\bar{v}t)$ where $\bar{v}=\bar{p}/p_0$.

We remark that

$$n(p;0)=\omega(a^*(p)a(p))$$

where ω is a state localised in the neighbourhood of the point $(0,0)$ of the coordinate system.

Then

$$n(p;b)=\omega(\Gamma_b(a^*(p)a(p))).$$

By (10) one gets

$$\begin{aligned}n(p;b)&=n(p;0)\exp(-\gamma bp)\\ &=n(p;0)\exp[-\gamma(tp_0-\bar{p}^2t/p_0)]\\ &=n(p;0)\exp[-\gamma(m^2/p_0)t].\end{aligned}$$

Therefore the decay time of the moving particles is given by

$$\tau=p_0/\gamma m^2=(p_0/m)\tau_0 \tag{11}$$

where $\tau_0=1/\gamma m$ is the decay time in the rest frame of the particles. As predicted by dilatation arguments the decay time increases with the energy.

This result is well known and can be obtained by those representations of the Poincaré semigroup $\mathscr{F}$ on the level of the one-particle space [1, 4].

The next application is one which cannot be obtained by representations restricted to the one-particle level. It is a genuine consequence of our representations in a second quantised theory.

4.2. *Particle number fluctuations*

We compute the evolution of the number of particles for an initial n-particle state under the stochastic semigroup evolution which we found above. In particular let ψ_n be the n-particle state

$$\psi_n = \frac{1}{(n!)^{1/2}} a^*(\phi)^n \Omega_{\mathrm{F}}$$

where Ω_{F} is the Fock vacuum, ϕ is any test function and

$$N(f) = \int f(p) a^*(p) a(p) \, \mathrm{d}\mu_m(p)$$

the number operator weighted with the function f.

Now we compute the characteristic function of the number operator $N(1)$ in the state ψ_n after applying Γ_b.

Using the following formula

$$\sum_{k=0}^{\infty} \int \prod_{j=1}^{k} \frac{[\exp(\mathrm{i}sf(p_j)) - 1] \, \mathrm{d}\mu_m(p_j)}{k!} \prod_{j=1}^{k} a^+(p_j) \prod_{j=1}^{k} a(p_j)$$
$$= \exp \mathrm{i}sN(f)$$

and (10), one obtains

$$\Gamma_b(\exp \mathrm{i}sN(1))$$
$$= \sum_{k=0}^{\infty} \int \prod_{j=1}^{k} \frac{\exp(-\gamma b p_j)(\mathrm{e}^{\mathrm{i}s} - 1) \, \mathrm{d}\mu_m(p_j)}{\kappa!} \prod_{j=1}^{k} a^+(p_j) \prod_{j=1}^{k} a(p_j)$$
$$= \exp N(g_{s,b})$$

where $g_{s,b}(p) = \ln[1 + \exp(-\gamma bp)(\mathrm{e}^{\mathrm{i}s} - 1)]$.

Suppose now that we take for $|\phi|^2$ a δ-function convergent sequence tending to $\delta(\cdot - p)$; then

$$(\psi_n, \Gamma_b(\exp \mathrm{i}sN(1))\psi_n)$$

tends to

$$\exp\{n \ln[1 + \exp(-\gamma bp)(\mathrm{e}^{\mathrm{i}s} - 1)]\} = [\eta + (1 - \eta) \, \mathrm{e}^{\mathrm{i}s}]^n$$

where $\eta = 1 - \exp(-\gamma bp)$ is the probability of decay.

Our remarks on the statistical independence of the decay processes described by the binomial distribution of order n for an n-particle state.

Now we consider the distribution of the decay products

$$(\psi_n, \Gamma_b(\exp \mathrm{i}s(N(1) - n))\psi_n).$$

This becomes

$$[e^{-is}\eta + (1-\eta)]^n.$$

If we now take γ to be small then this becomes

$$\left(1+\frac{n\gamma bp}{n}(e^{-is}-1)\right)^n.$$

If one takes $n\gamma bp \simeq O(1)$ and n large enough then one recovers the characteristic function of a Poisson distribution.

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